International Journal of Applied Mathematics & Statistical Sciences (IJAMSS);
ISSN (P): 2319–3972; ISSN (E): 2319–3980
Vol. 14, Issue 2, Jul–Dec 2025; 21–36

International Academy of Science, Engineering and Technology
IASET
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APPLICATIONS OF FRACTIONAL DOMINATING NUMBER

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ABSTRACT

The fractional domination number, a graph theory concept, generalizes the traditional domination number by assigning fractional weights (ranging from 0 to 1) to vertices in a dominating set, typically optimized using linear programming. This approach is valuable for optimization challenges requiring partial coverage or efficient resource allocation. Here, I outline its applications in Wireless Sensor Networks (WSN), Image Compression, and Metro Train Optimization, Optimization of Intercity Bus Networks based on recent research.

KEYWORDS: *Wireless Sensor Network, Fractional Domination Number, LP Formulation, Dijkstra'salgorithms.*

Article History

***Received:** 16 Sep 2025 | **Revised:** 18 Sep 2025 | **Accepted:** 22 Sep 2025*

INTRODUCTION

Let G = (V, E) be a graph. A subset D⊆V is called a dominating set of graph G if each vertex in V - D is adjacent to at least one vertex in D. The size of the smallest dominating set determines a graph's domination number. The fractional domination number, on the other hand, allows for a real-valued weight to be assigned to each vertex in the graph. A fractional dominating set assigns non-negative weights to vertices such that the sum of the weights of a vertex and its neighbors is at least one for every vertex. The fractional domination number of a graph is defined as follows: A function f: V(G) → [0,1] is referred to as a fractional dominating function (FDF) if $\sum_{v_1 \epsilon N[v]} f(v_1) \geq 1$ for every vertex v in V(G), where N[v] is the closed neighborhood of v. The minimum weight among all such FDFs of G is recognized as the fractional domination number (FDN), denoted by γ_f(G) [2].

Fisher [6] has given a relation between the fractional dominating number of G and the fractional dominating number of its graph complement. They have defined the fractional domination number using linear programming, where 1 is the vector of all ones and 0 is the vector of all zeros, A(G) is the adjacency matrix of the graph, and I is the identity matrix. The fractional dominating number gamma(G) is the objective function value of the linear programming problem:

Minimize 1^T x,

Subject to constraints (A(G)+ I)x ≥ 1, and x ≥ 0., The solution to this equation provides the fractional dominating number with non-negative weights on vertices whose sum in any closed neighborhood is at least one. By forcing x to have integer entries, the equation transforms into an integer program for the domination number of the graph. A similar formulation for the fractional total dominating number requires the sum in any open neighborhood to be at least one. By forcing integer entries, it transforms into an integer program for the total domination number of the graph.

In image processing, the fractional domination number can be applied to tasks such as image compression and reconstruction, particularly for images with fractal-like self-similar structures. By modeling an image as a graph where pixels are vertices and edges represent spatial proximity or color similarity, the fractional domination approach identifies a minimal weighted set of critical pixels that "dominate" the entire image. This allows for compressing the image to 20-30% of its original pixels while enabling accurate reconstruction through interpolation. For instance, the total regular fractional domination number optimizes pixel selection, reducing data storage and transmission costs without significant loss in quality.

In metro station optimization, the fractional domination number on litact graph offers a framework for efficient resource allocation in transportation networks modeled as graphs, where stations are vertices and rail lines are edges. This concept can be used to minimize the number of key stations requiring monitoring, signaling upgrades, or maintenance crews while ensuring full network coverage. For vulnerability analysis in metro systems, parameters derived from domination (such as edge-domination and bondage numbers) help identify critical points prone to disruptions, and the fractional variant allows for weighted, partial resource deployment to optimize costs and improve reliability in high-demand urban rail networks.

Public transportation plays a key role in regional development by providing affordable mobility and reducing congestion, fuel use, and environmental impact. Designing an efficient bus network requires both full coverage of towns and minimal travel time between destinations. While shortest-path algorithms such as Dijkstra's are effective for route planning, they do not solve the larger problem of selecting transfer hubs. Using graph theory, the concept of fractional domination enables flexible and cost-efficient hub placement while ensuring every town lies within a defined coverage radius. Once hubs are chosen, shortest-path algorithms identify the most efficient hub-to-hub corridors, forming the backbone of the network. Modeling the intercity bus system of Tamil Nadu as a weighted graph, this combined method shows how graph-theoretic optimization can improve network design, making it more reliable, economical, and passenger-friendly.

The second section contains basic concepts of dominating set, domination number, fractional domination number, and wireless sensor networks. The third section includes an algorithm for computing the fractional dominating number in wireless sensor networks using the LPP formulation. In fourth section example problem for wireless sensor network . In fifth section image processing and progression and in sixth session transportation and metro train optimization. Finally in seventh session Optimization of Intercity Bus Networks based on recent research.

BASIC DEFINITIONS

Definition

Let G = (V,E) is graph with V as vertex set and E as edge set. A subset D of V is called a dominating set of graph G if each vertex in V − D is connected to at least one vertex in D. For every vertex u ∈ V −D, d(u,D) = 1 or N[D] = V . A domination set D is known as minimal dominating set (MDS) if no proper subset of D is a dominating set of G. The size of smallest dominating set of graph G is called the domination number of graph G and it is denoted by γ(G). The maximum cardinality of minimal dominating set (MDS) of graph G is called upper domination number of G and it is denoted by γ(G). The domination number noted by γ(G) and the upper domination number noted by γ(G) are defined as:

$$\gamma(G) = \min\{|D| : D \text{ is MDS of } G\} \text{ and } \Gamma(G) = \max\{|D| : D \text{ is MDS of } G\}.$$

Definition

A dominating function f to be any f : V(G) → [0,1] is function of G which allocates the values for each vertex v ∈ V(G) in the unit interval [0,1]. The given function f is called a fractional dominating function if for every vertex v ∈ V(G), f (N[v]) = R v∈N[v] f (v) ≥ 1. It denotes the total value of the vertices in the closed neighborhood of v ∈ V(G) such that N[v] is at least one, i.e., (N[v]) ≥ 1 (since then any vertex v ∈ V(G) is in the closed neighborhood of at least one vertex in D, where D is subset of vertex set V). For any v ∈ V(G) , define a function f : V(G) → [0,1] by $f(v)=\sum_{v_1 \epsilon N[v]} f(v_1) \geq 1$,for all v ∈ V(G) and f is a fractional dominating function.

Definition

The dominating function f is called a minimal fractional dominating function (MFDF) if there does not exist a dominating function g≠ f for which g(v) ≤ f (v) for all v ∈ V(G) equivalently f is an minimal fractional dominating function (MFDF) if for every vertex v with f (v) > 0, there exist a vertex w ∈ N[v] such that $\sum_{v_1 \epsilon N[v]} f(v_1)= 1$. If there is a vertex v for which given condition is not true means every vertex in the closed neighborhood of v obeys (N[v]) > 1, then we can decrease f(v) to obtain a smaller fractional dominating function and so f is not a minimal fractional dominating function (MFDF).

The fractional domination number of G denoted by $\gamma_f(G)$ and the upper fractional domination number of G denoted by Γ_f (G) are defined as,

$$\gamma_f(G)= \min\{|f| : f \text{ is an MFDF of } G\}, \Gamma f (G) = \max\{|f| : f \text{ is an MFDF of } G\},$$

Where $|f| = \sum_{v \epsilon G} f(v)$. Fractional domination number as 1 be the vector of all ones. Let 0 be the vector of all zeros. Let A(G) be the adjacency matrix. The fractional domination number $\gamma_f(G)$ is the value of linear program:

Objective function minimize $Z= 1^T x$,

Subject to conditions $(A(G)+ I)x \geq 1$, and $x \geq 0$.

Definition

Wireless Sensor Networks (WSNs) represent a decentralized wireless architecture characterized by the deployment of numerous sensor nodes in an ad-hoc configuration, designed to monitor various system, physical, or environmental conditions. These sensor nodes, equipped with onboard processors, facilitate the management and observation of the surrounding environment within a designated area. They connect to a Base Station, which serves as the processing unit within the WSN framework. The Base Station is further linked to the Internet, enabling the dissemination of data. WSNs are adept at processing, analyzing, storing, and mining data, thereby enhancing the overall efficacy of information management.

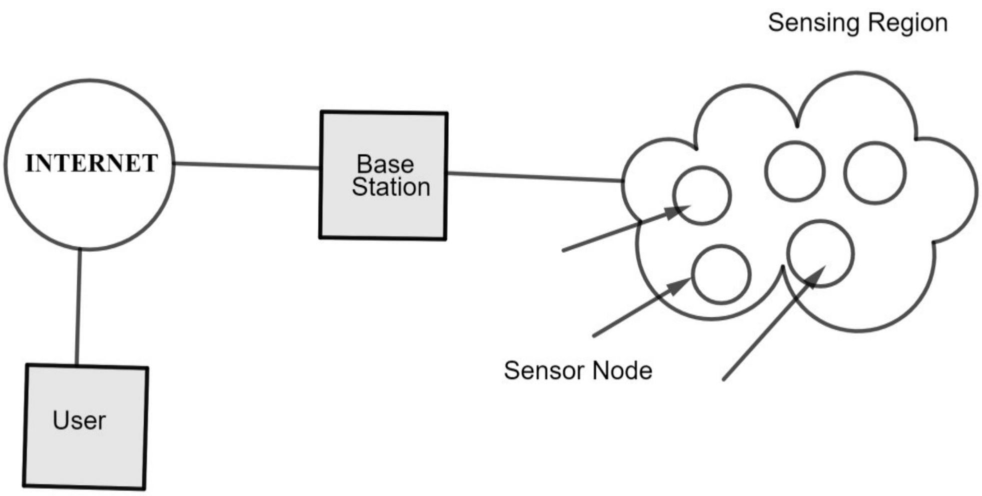
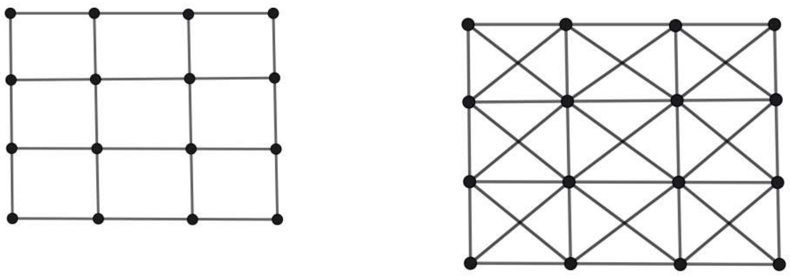


Figure 1

Pixel Connected to Graph

Pixel Adjacency Graph-2D



4 Connected Pixel Adjacency Graph **8 Connected Pixel Adjacency Graph**

Figure 2

Definition

The working theory of GPS using Graph theory we could find the domination which is meant to be as a shortest path to reach our destination. The most used algorithm in GPS to find the domination is Dijkstra's algorithm.

Dijkstra's algorithm is a graph search algorithm that finds the shortest path from a source vertex to all other vertices in a weighted graph. It is widely used in network routing and mapping applications.

Graph Type: Works on graphs with non-negative weights.

Shortest Path: Finds the minimum cost path from the source to each vertex.

Greedy Approach: Expands the shortest path by selecting the closest vertex at each step.

How It Works:

- Initialization: Set the distance to the source vertex as 0 and all other vertices as infinity. Mark all vertices as unvisited.
- Vertex Selection: Choose the unvisited vertex with the smallest tentative distance. Mark it as visited.
- Update Distances: For each neighbour of the current vertex, calculate the tentativedistance. If this path is shorter than the known distance, update it.
- Repeat: Continue until all vertices have been visited or the smallest tentative distance is infinity (indicating unreachable vertices).
- Time Complexity: Using a simple array: O (V_2) (where V is the number of vertices).Using a priority queue (heap): O((V+E)logV) (where E is the number of edges).

PROBLEM RELATED TO WIRELESS STATEMENT

Consider a region that needs to be monitored using wireless camera sensors. Each sensor can monitor its own location and its neighboring nodes with in sensing range. Then to minimize the number of active sensors while ensuring full coverage of the area. Here using fractional domination to assign partial monitoring responsibility to multiple sensors.

Algorithm for Construct LP Formulation from Wireless Sensor Network to Graph

- Construct the given sensors as a vertex and communications as an edge.
- To develop a fractional domination formulation.
- Minimize our number of sensors that is our target.
- In each sensor identifying closed neighborhood N[s]
- Formulate LP form satisfying fractional domination condition.
- Using Linear Programming solver finding the solution.

Algorithm to Find Fractional Dominating Number

- Let x_i be non-negative decision variables representing the fraction of vertices in the dominant set that include vertex i.
- The objective function is to Minimize Z = $f(x_1)+f(x_2)+...+f(x_n)$, which represents the size of the dominating set.
- The constraints are:
  - For every vertex i, the sum of the fractions of its neighbor that are in the dominating set must be at least 1 $\sum_{j \in N(i)} f(x_j) \geq 1$, for all i in G
  - Each variable $f(x_i) = \frac{1}{1+k} = Y_i$ is feasible to LPP and yields the same objective function value of $\frac{n}{1+k}$, where Each variable $f(x_i)$ must be non-negative $x_i \geq 0$, for all i = 1,2,...,n.
- Solve the resulting LPP using a linear programming solver to obtain the fractional domination number of the graph G. Using a linear programming solver, we obtain the optimal solution: xi = n/k + 1,

This solution corresponds to the set of vertices in G, where each vertex i is assigned a fraction of (1/k +1). S. The sum of the fractions of the vertices in the dominating set is equal to the size of the dominating set, which is the objective function that we are minimizing. Therefore, the fractional domination number of the regular graph G is 1, which is achieved by choosing each vertex with a fraction of (1/k + 1) in the dominating set and graph G has n vertices with (n−1)-regular graph.

EXAMPLE PROBLEM

- Consider 6 sensor as nodes (6) and edges represents communication (7) convert into Graph G.

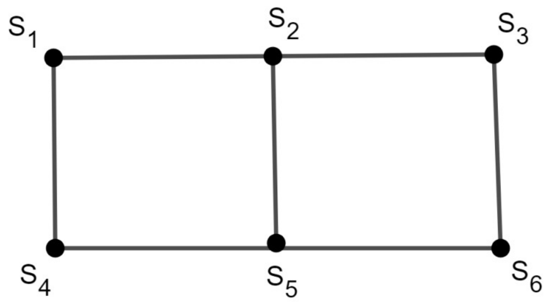


Figure 3

- Let f : S→{0,1} be a function of a graph G=(S,σ,μ) in figure with vertex V={S_1,S_2,S_3,S_4, S_5,S_6}, E={S_1S_2, S_1S_4,$S_2S_5S_2S_3$,S_3S_6,S_4,S_5S_6}
- $N_G[S_1]=\{S_1,S_2,s_4\}$,
  $N_G[S_2]=\{S_1,S_2,S_3,S_5\}$,$N_G[S_3]=\{S_2,S_3,S_6\}$,$N_G[v_4]=\{S_1,S_4,S_5\}$,$N_G[S_5]=\{S_2,S_4,S_6,S_5\}$, $N_G[S_6]=\{S_5,S_3,S_6\}$f is dominating function of G.

Find the Domination Number $\gamma_f(G)$ in Figure by Solving the Following Linear Programming Problem Which is Formulated using G

Let f_1,f_2,f_3,f_4,f_5,f_6 be the fractional weights of S_1,S_2,S_3,S_4, S_5,S_6

Minimize $z = f_1+f_2+f_3+f_4+f_5+f_6$

Subject to $f_1+f_2+f_4 \geq 1$

$f_1+f_2+f_3+f_5 \geq 1$

$f_2+f_3+f_6 \geq 1$

$f_1+f_4+f_5 \geq 1$

$f_2+f_4+f_5+f_6 \geq 1$

$f_3+f_5+f_6 \geq 1$

Solution: Minimize $z = f_1+f_2+f_3+f_4+f_5+f_6$

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{pmatrix} \geq \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$ and $f(s_i) \epsilon\ [0,1]$ for all $s_i \epsilon\ V(G)$ where

N= $\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$, $X_{fD} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{pmatrix}$ $\overrightarrow{1_n} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ be the column vector with respect to the constraint part of L.P.P.

The optimal solution of the above L.P.P is Minimize Z= 6*1/3=2

where f_1=1/3, f_2=1/3, f_3 =1/3, f_4=1/3, f_5=1/3, f_6 =1/3

Hence the fractional domination number γ_f(G)= 2.

So that using fractional domination number 2 sensors only ON to cover all areas that is 2 sensors worth of energy is needed, by having each sensor ON 33% of the time. So that extends the network life time (battery savings).

IMAGE PROCESSING AND TRANSFORMATION

Image processing and transformation involve modifying images to enhance quality and standardize features. These techniques improve the efficiency and accuracy of models by ensuring uniformity in image data. Since images can vary in size, lighting, contrast, and noise levels, preprocessing ensures consistency and reduces unwanted variations.

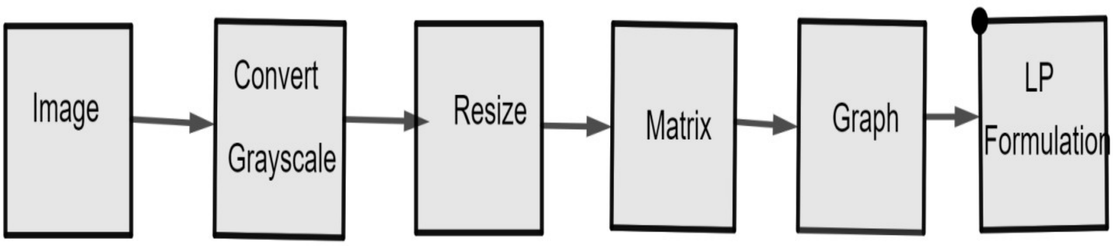


Figure 4

Grayscale Image

Grayscale image is one of the digital image categories where every pixel may only be of varying shades of grey without any color information. The process of transforming three plane color data into single plane is known as color-to-grayscale image conversion [13].

In a grayscale image, every pixel digitized can hold an intensity value of brightness for the grey shade in consideration. In most cases, these range from 0 – 255 in an 8-bit grayscale image whereby 0 is represented by black, 255 by white and all other values lies between the two extremes as grey. For instance, while working with 8-bit image, the intensity of the grey level can only have 256 values, whereas, the 16-bit image provides a much wider array of intensities with 65, 536 possible intensities. Grayscale images are useful for image processing and computer visions, as well as any other applications that require less computational power than for dealing with colors images. Although they are superior in most other aspects, they are especially important in situations when color information is not required in the first place, thus saving more space and processing time.

Resizing the Image

Resizing an image is the process of changing its dimensions (height, width) while maintaining its essential features. It involves either increasing or decreasing the number of pixels in the image. Resizing is crucial in various application such as image processing, computer vision, machine learning where standardizing image is necessary.

Resizing the image is very important because of memory optimization and improved processing speed and aspect ratio preservation. Resized image gave different dimensions and pixel count compared to the original image. And also smaller file size making is efficient for storage, transmission and faster processing.

Convert to Matrix

An image is an essentially a grid of pixels, where each pixel has a value representing its color or intensity. This grid can be represented as a matrix(2D for grayscale and 3D for color images with channels like RGB)

Resized image = Matrix representation of pixels

It Is because when we resize an image, we are altering the pixel grid, but we still need to work with the image as a matrix for further analysis, feature extraction and image processing tasks. This conversion is essential for further processing or applying algorithms like feature extraction, classification, etc. Each pixel in the resized image is transformed into a matrix element, and the whole image becomes a matrix.

Image Constructed as a Graph

Construct graph G=(V, E) where each pixel is a vertex and edges via adjacency. Then weights edges by intensity or color differences.

Formulate LP Model

The above graph is converted to Linear Programming model.

Objective Function is Minimize Z= f (x_1)+$f(x_2)$+...+$f(x_n)$,

Subject to the constraints $\sum_{j\epsilon N(i)} f(x_j) \geq 1$, for all j in G .

Image Compression using LP Formulation

Minimize z =$\sigma_D(v_1)$ +$\sigma_D(v_2)$ +$\sigma_D(v_3)$ +$\sigma_D(v_4)$

Subject to $\sigma_D(v_1)$+$\sigma_D(v_2) + \sigma_D(4)$ $\geq$1,$\sigma_D(v_1)$+$\sigma_D(v_2) + \sigma_D(v_3)$ $\geq$1,$\sigma_D(v_2)$+$\sigma_D(v_3) + \sigma_D(v_4)$ $\geq$1, $\sigma_D(v_1)$+$\sigma_D(v_3) + \sigma_D(v_4)$ $\geq$1 and $0 \leq \sigma_D(v_i) \leq 1$ for all V(G_D)

Solution: Minimize z $=\sigma_D(v_1)+\sigma_D(v_2)+\sigma_D(v_3)+\sigma_D(v_4)$

$$\begin{pmatrix}1 & 10 & 1\\ 1 & 11 & 0\\ 0 & 11 & 1\\ 1 & 01 & 1\end{pmatrix}\begin{pmatrix}\sigma_D(v_1)\\ \sigma_D(v_2)\\ \sigma_D(v_3)\\ \sigma_D(v_4)\end{pmatrix}\geq\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$ and $\sigma_D(v_i)\epsilon$ [0,1] for all $v_i\epsilon$ V(G) where

N= $\begin{pmatrix}1 & 10 & 1\\ 1 & 11 & 0\\ 0 & 11 & 1\\ 1 & 01 & 1\end{pmatrix}$, $\boldsymbol{X_{fD}}$=$\begin{pmatrix}\sigma_D(v_1)\\ \sigma_D(v_2)\\ \sigma_D(v_3)\\ \sigma_D(v_4)\end{pmatrix}$ $\overrightarrow{\boldsymbol{1_n}}$ = $\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$ be the coloumn vector with respect to the constraint part of L.P.P.

Hence the image processing and compressing using Fractional domination number .It reduces image data based on structural dominance also allows fractional pixel influence get better quality.

TRANSPORTATION & METRO NETWORK OPTIMIZATION

Transportation optimization involves planning and managing the movement of people or goods in a way that maximizes efficiency and minimizes costs. It requires selecting the most effective routes, schedules, transportation modes, and resource allocations to shorten travel times, reduce expenses, and enhance service quality. In operations research, models like the transportation problem use linear programming to allocate shipments from sources to destinations at the lowest cost while satisfying supply and demand requirements.

Metro train optimization deals with enhancing the efficiency and effectiveness of urban rail networks. Scheduling optimization is creating train timetables that minimize passenger wait times and prevent overcrowding. Route optimization is determining the best service patterns and paths for maximum coverage with minimal travel times. Energy efficiency optimization is controlling acceleration, deceleration, and coasting to reduce energy usage. Capacity planning is deploying the right number of trains to meet passenger needs without excess. Infrastructure utilization is improving the use of tracks, stations, and signaling to avoid delays.In academic and practical applications, metro optimization is often approached using graph theory, shortest path methods, and linear programming, where stations are modeled as nodes, tracks as edges, and optimization methods are applied to improve passenger flow, shorten travel duration, and reduce operational expenses.

Steps for Optimization of Metro Train

- First consider metro model consider some smaller area is to metro graph
- Next convert to Litact graph
- Find Fractional domination number
- Interpret for optimization.

Example

Consider Chennai metro 8 station names. That are

- Washermanpet
- Royal Park

- Mannadi
- High Court
- Central
- Government Estate
- LIC
- Thousand Lights

Using these 8 stations to form a graph such as stations considered as vertices and consecutive track connections.

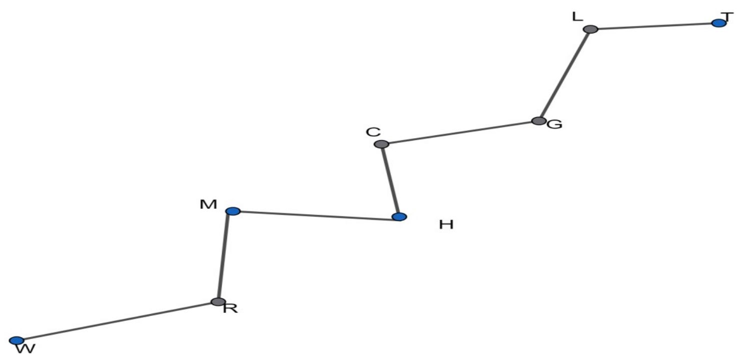


Figure 5

Convert to Litact Graph

In a Litact Graph, Original stations as vertices and each edge becomes a new vertex in that track section. Then edge connect an original vertex to an edge -vertex if the station is an endpoint of that track. So eight station vertices and seven edge vertices. Totally 15 vertices to form a litact vertices.

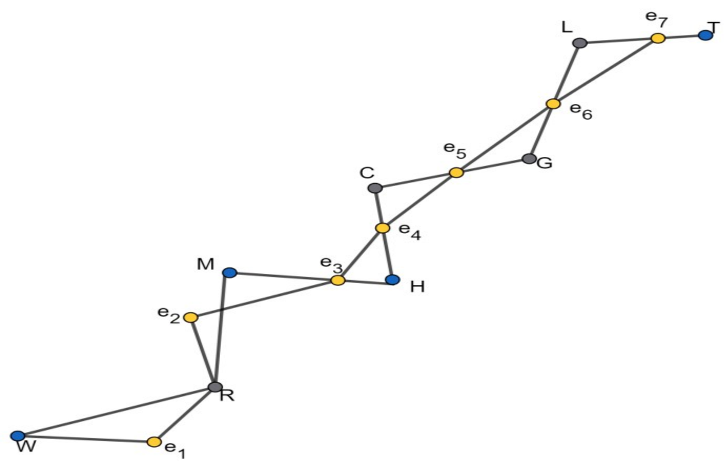


Figure 6

This is the Litact graph showing both stations (blue) and track segments (orange) as separate vertices.

LP Formulation

Let f : V→{0,1} be a function of a graph Gfor every vertex v϶ V(L(G) (stations +track)

The above graph is converted to Linear Programming model.

Objective Function is Minimize Z= f (x_1)+$f(x_2)$+...+$f(x_n)$,

Subject to the constraints $\sum_{j\epsilon N(i)} f(x_j) \geq 1$, for all j in G .

Find the Domination Number $\gamma_f(G)$ in Figure by Solving the Following Linear Programming Problem Which is Formulated using L(G)

Minimize z = $f_1+f_{e1}+f_2+f_{e2}+f_3+f_{e3}+f_4+f_{e4}+f_5+f_{e5}+f_6+f_{e6}+f_7+f_{e7}+f_8$

Subject to $f_W+f_{e1} \geq 1$

$f_W+f_{e1}+f_R \geq 1$

$f_R+f_{e1}+f_{e2} \geq 1$

$f_{e2}+f_R+f_M \geq 1$

$f_M+f_{e2}+f_{e3} \geq 1$

$f_{e3}+f_M+f_H \geq 1$

$f_H+f_{e3}+f_{e4} \geq 1$

$f_{e4}+f_H+f_C \geq 1$

$f_C+f_{e4}+f_{e5} \geq 1$

$F_{e5}+f_G+f_C \geq 1$

$f_G+f_{e5}+f_{e6} \geq 1$

$F_{e6}+f_G+f_L \geq 1$

$f_L+f_{e6}+f_{e7} \geq 1$

$F_{e7}+f_L+f_T \geq 1$

$f_T+f_{e7} \geq 1$

Solution

Minimize z = $f_1+f_{e1}+f_2+f_{e2}+f_3+f_{e3}+f_4+f_{e4}+f_5+f_{e5}+f_6+f_{e6}+f_7+f_{e7}+f_8$

$$\begin{pmatrix} 1 & 1 & 00 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 0 \\ 1 & 1 & 10 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 1 & 11 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 11 & 1 & 00 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 01 & 1 & 10 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 1 & 11 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 11 & 1 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 01 & 1 & 10 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 1 & 11 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 11 & 1 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 01 & 1 & 10 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 00 & 1 & 11 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 11 & 1 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 01 & 1 & 1 \\ 0 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 00 & 1 & 1 \end{pmatrix} \begin{pmatrix} f_1 \\ f_{e1} \\ f_2 \\ f_{e2} \\ f_3 \\ f_{e3} \\ f_4 \\ f_{e4} \\ f_5 \\ f_{e5} \\ f_6 \\ f_{e6} \\ f_7 \\ f_{e7} \\ f_8 \end{pmatrix} \geq \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } f(V) \epsilon [0,1] \text{ for all}$$

V ϵ L(G) where

$$N = \begin{pmatrix} 1 & 1 & 00 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 0 \\ 1 & 1 & 10 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 1 & 11 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 11 & 1 & 00 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 01 & 1 & 10 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 1 & 11 & 0 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 11 & 1 & 00 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 01 & 1 & 10 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 1 & 11 & 0 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 11 & 1 & 00 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 01 & 1 & 10 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 00 & 1 & 11 & 0 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 11 & 1 & 0 \\ 0 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 01 & 1 & 1 \\ 0 & 0 & 00 & 0 & 00 & 0 & 00 & 0 & 00 & 1 & 1 \end{pmatrix}, X_{fD} = \begin{pmatrix} f_1 \\ f_{e1} \\ f_2 \\ f_{e2} \\ f_3 \\ f_{e3} \\ f_4 \\ f_{e4} \\ f_5 \\ f_{e5} \\ f_6 \\ f_{e6} \\ f_7 \\ f_{e7} \\ f_8 \end{pmatrix} \& \overrightarrow{\mathbf{1}_n} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

The optimum solution of this equations are

$$f_1 = 0, f_{e1} = 1, f_2 = 0, f_{e2} = 0, f_3 = 1, f_{e3} = 0, f_4 = 0, f_{e4} = 1, f_{5=0}, f_{e5} = 0, f_6 = 1, f_{e6} = 0, f_7 = 0 f_{e7} = 1, f_8 = 0$$

The objective value of Fractional domination number is γ_f(L(G)) = 5

In this paper the 8-station stretch of the Chennai Metro (Washermanpet to Thousand Lights) was represented as a Litact graph, where both stations and track segments are treated as vertices. This produced a 15-vertex alternating path structure.

Indicating that the entire network can be covered using the equivalent of five full monitoring/maintenance resources. One possible optimal placement is at E1 (Wash–Royal), Mannadi, E4 (High Court–Central), Government Estate, and E7 (LIC–Thousand Lights), each covering itself and its two adjacent vertices to ensure complete coverage.

Since the solution is already integral, fractional scheduling does not further reduce the required number; five is both the minimum possible (fractional bound) and a feasible integer deployment. positioning five units at these sites—or an equivalent alternative arrangement—minimizes deployment costs while ensuring full station and track coverage. If certain sites are unsuitable, symmetrical shifts along the path can provide equivalent solutions.

USING FRACTIONAL DOMINATION METHOD TO FIND SHORTEST DISTANCE BASED ON GPS ROUTES

Hosur to maduraithere are 3 routes based on GPS to reach our destination Madurai using fractional domination we can find the shortest distance to reach using graph theory the diagram has drawn (fig 5.6).based on edge and the vertices we could find the shortest distance where GPS tracking will also play an crucial role as dijkstras algorithm.

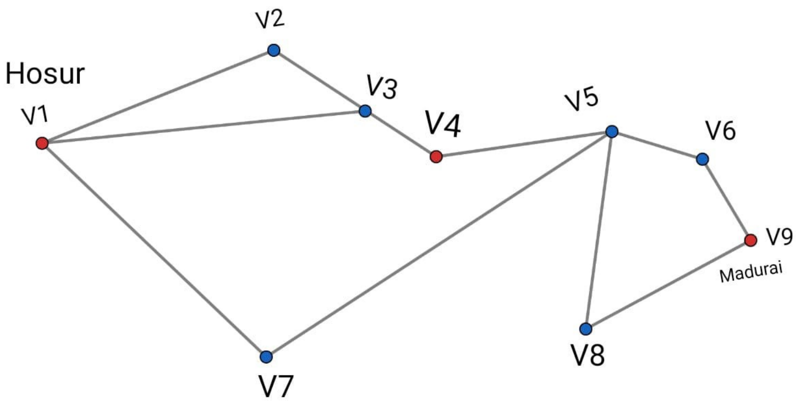


Figure 7

Simplex Big-M method LPP is used to find the shortest distance which come under the fractional domination type. As per the vertex , nodes and weight based on this we can form the diagram points $V_{1 \ldots\ldots\ldots\ldots} V_9$ denotes the places where V_1 is the starting point (hosur) and our destination is V_9 (Madurai).

Table 1

Edge	Weight (c_{ij})
$V_1 \rightarrow V_2$	1
$V_1 \rightarrow V_7$	2
$V_2 \rightarrow V_3$	2
$V_2 \rightarrow V_5$	2
$V_3 \rightarrow V_4$	1
$V_4 \rightarrow V_5$	2
$V_5 \rightarrow V_6$	1
$V_5 \rightarrow V_7$	1
$V_6 \rightarrow V_9$	3
$V_7 \rightarrow V_8$	2
$V_8 \rightarrow V_9$	1

Find Solution using Simplex Method (Big M Method)

MIN Z = $X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9$

CONSTRAINS

$X_1+X_2+X_7 \geq 1$

$X_1+X_2+X_3 \geq 1$

$X_2+X_3 \geq 1$

$X_3+X_4+X_5 \geq 1$

$X_4+X_5+X_6+X_7 \geq 1$

$X_5+X_6+X_9 \geq 1$

$X_1+X_7+X_8 \geq 1$

$X_7+X_8+X_9 \geq 1$

$X_6+X_8+X_9 \geq 1$

And $X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9 \geq 0$

Solution

Problem is

MIN Z = $X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9$

Subject to

$X_1+X_2+X_7 \geq 1$

$X_1+X_2+X_3 \geq 1$

$X_2+X_3 \geq 1$

$X_3+X_4+X_5 \geq 1$

$X_4+X_5+X_6+X_7 \geq 1$

$X_5+X_6+X_9 \geq 1$

$X_1+X_7+X_8 \geq 1$

$X_7+X_8+X_9 \geq 1$

$X_6+X_8+X_9 \geq 1$

And $X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9 \geq 0$

The problem is converted to canonical from by adding slack, surplus and artificial variables as appropriate

- As the constraint -1 is of type '≥' we should subtract surplus variable S_1 and add artificial variable A_1
- As the constraint -2 is of type '≥' we should subtract surplus variable S_2 and add artificial variable A_2
- As the constraint -3 is of type '≥' we should subtract surplus variable S_3 and add artificial variable A_3
- As the constraint -4 is of type '≥' we should subtract surplus variable S_4 and add artificial variable A_4
- As the constraint -5 is of type '≥' we should subtract surplus variable S_5 and add artificial variable A_5
- As the constraint -6 is of type '≥' we should subtract surplus variable S_6 and add artificial variable A_6
- As the constraint -7 is of type '≥' we should subtract surplus variable S_7 and add artificial variable A_7
- As the constraint -8 is of type '≥' we should subtract surplus variable S_8 and add artificial variable A_8
- As the constraint -9 is of type '≥' we should subtract surplus variable S_9 and add artificial variable A_9

After Introducing Surplus, Artificial Variables

Min Z = $X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+0S_1+0S_2+0S_3+0S_4+0S_5+0S_6+0S_7+0S_8+0S_9$

$+MA_1+MA_2+MA_3+MA_4+MA_5+MA_6+MA_7+MA_8+MA_9$

$X_1+X_2+X_7-S_1+A_1=1$

$X_1+X_2+X_3-S_2+A_2=1$

$X_2+X_3-S_3+A_3=1$

$X_3+X_4+X_5-S_4+A_4=1$

$X_4+X_5+X_6+X_7-S_5+A_5=1$

$X_5+X_6+X_9-S_6+A_6=1$

$X_1+X_7+X_8-S_7+A_7=1$

$X_7+X_8+X_9-S_8+A_8=1$

$X_6+X_8+X_9-S_9+A_9=1$

$X_1,X_2,X_3,X_4,X_5,X_6,X_7,X_8,X_9,S_1,S_2,S_3,S_4,S_5,S_6,S_7,S_8,S_9,A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9 \geq 0$

Since all $Z_j - C_j \leq 0$, Hence, optimal solution is arrived with value of variables as :

X_1= 1\2 , X_2 = 0 , X_3 = 1, X_4 = 0, X_5 = 0, X_6 =1\2, X_7 = 1\2, X_8 = 0, X_9= 1\2

min z = 3

Final Shortest Path Using the Fractional Dominating Set

PATH: $X_1 \rightarrow X_3 \rightarrow X_6 \rightarrow X_7 \rightarrow X_9$

$V_1 = X_1, V_2 = X_2, V_3 = X_3, V_4 = X_4, V_5 = X_5, V_6 = X_6, V_7 = X_7, V_8 = X_8, V_9 = X_9$

HOSUR→KELAMANGALAM→KARUR→DINDIGUL→MADURAI

TOTAL DISTANCE : 380 km

CONCLUSION

Fractional domination provides a common mathematical strategy for achieving optimal coverage in various domains such as WSNs, image processing, and transportation networks. It effectively minimizes cost while maintaining performance standards. Going forward, research will focus on enhancing the method to handle priority-based vertex weighting, changing network topologies, and fault resilience. Such improvements could expand its practical use in emerging areas like urban infrastructure management, medical sensor systems, and advanced mobility planning.

Also this project explores the use of graph theory in GPS-based optimization, highlighting domination concepts for efficient route planning. Fundamental ideas such as simple graphs, isomorphism, and domination sets were introduced to enhance coverage and connectivity. Unlike LLP-domination, the fractional domination number was applied to reduce computations while ensuring effective routing. The shortest path problem was modeled as a Linear Programming task and solved with the Simplex method for improved accuracy. Overall, the integration of graph theory, fractional domination, and LP provides a powerful framework for intelligent transportation and urban planning.

REFERENCES

1. *Mahesh Sarada , Rekha Jain and Ganesh Mundhe, 'Algorithm Using LPP for Domination and Fractional Domination Number of Some Graphs' Communications in Mathematics and Applications. ISSN 0975-8607,DOI: 10.26713/cma.v15i5.2840*
2. *I. F. Akyildiz and I. H. Kasimoglu, "Wireless sensor and actor networks: Research challenges," Ad Hoc .Netw., vol. 2, no. 4, pp. 351–367, May 2004.*
3. *D. C. Fisher, Fractional dominations and fractional total dominations of graph complements,Discrete Applied Mathematics 122 (1-3) (2002), 283 – 291, DOI: 10.1016/s0166-218x(01)00305-5.*
4. *G. H. Fricke, M. A. Henning, O. R. Oellermann and C. S. Henda, An algorithm to find two distance domination parameters in a graph, Discrete Applied Mathematics 68(1-2) (1996), 85 – 91, DOI: 10.1016/0166-218x(95)00057-x*
5. *M.Meenakshi and V.Pankajam, Dominating Function in Fractional Directed Graph, Indian Journal of Natural Science, 15(85), 2024, 63326-63332. (Web of Science)*
6. *M. Meenakshi and V.Pankajam, Fractal Graph of Total Regular Fractional Domination Number, Advances in Nonlinear Variational Inequalities, Vol 28 No. 3s (2025), 573-580 DOI:https://internationalpubls.com/index.php/anvi.*
7. *Meenakshi. M, & Pankajam. V. (2025). Exploring the Parameters of Fractional Domination Number in Litact Graph. Utilitas Mathematica, 122(1), 2701–2709. Retrieved from https://utilitasmathematica.com/index.php/Index/article/view/2568*